Functional Equivalence of Spectral Vegetation Indices

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Numerous formulae, vegetation indices, have been developed to reduce multispectral scanner (MSS) data to a single number for assessing vegetation characteristics such as species, leaf area, stress, and biomass. Part I of this report gives the history and formulae of some four dozen vegetation indices. Studies investigating the empirical relationships among vegetation indices are summarized. Part II of this report develops the idea of two vegetation indices being functionally equivalent: Two vegetation indices are taken to be equivalent for making a set of decisions, if the decisions made on the basis of one index could have been equally well made on the basis of the other index. The utility of these ideas is explored in the context of alarm models and graphical displays. Several widely used indices are shown to be equivalent.

Introduction

> The aim of science is to seek the simplest explanation of complex facts. We are apt to fall into the error of thinking that the facts are simple because simplicity is the goal of our quest. The guiding motto in the life of every natural philosopher should be, "Seek simplicity and distrust it."
> Alfred North Whitehead

Current and accurate information on a global basis regarding the extent and condition of the world's major food and fiber crops is important in today's complex world. Traditional sampling techniques for estimating crop conditions, based on field collection of data, are time-consuming, costly, and not generally applicable to foreign regions. An alternate approach is remote sensing. A series of earth observation satellites (Landsats) have provided a potential way to monitor worldwide crop conditions (MacDonald and Hall, 1980). The sensor system onboard the Landsats, the multispectral scanner (MSS), measures the reflectance of the scene in four wavelength intervals (channels) in the visible and near-infrared portions of the spectrum. The spectral measurements are influenced by the vegetation characteristics, soil background, and atmospheric condition.

Investigators have developed techniques for qualitatively and quantitatively assessing the vegetative canopy from spectral measurements. The objective has been to reduce the four channels of MSS data to a single number for predicting or assessing such canopy characteristics as leaf area, biomass, and percent ground cover.

This paper summarizes and references the origin, derivation, and motivation for some four dozen of these formulae which are referred to as vegetation indices (VIs). Part II develops the idea of two VIs being functionally equivalent for decision making. The meaning and utility of VIs equivalence is demonstrated in a sequence of real and hypothetical examples.

Development of Vegetation Indices

Idealized reflectance patterns for herbaceous vegetation and soil are compared in Fig. 1. Dead or dormant vegetation has higher reflectance than living vegetation in the visible spectrum and lower reflectance in the near-infrared. Soil has higher reflectance than green vegetation and lower reflectance than dead vegetation in the visible, whereas, in the near-infrared, soil typically has lower reflectance than green and dead vegetation (Tappan, 1980). Jackson et al. (1980), Tucker and Miller (1977), and Deering et al. (1975) provide an extensive discussion of reflectance properties.

Numerous vegetation indices have been used to make quantitative estimates of leaf area index, percent ground cover, plant height, biomass, plant population, and other parameters (Pearson and Miller, 1972 and Wiegand et al., 1974). Most formulae are based on ratios or linear combinations and exploit differences in the reflectance patterns of green vegetation and other objects as summarized in Fig. 1.

The digital counts (DCs) from the individual MSS channels (CH4, CH5, CH6, CH7) have been used to estimate percent ground cover and vegetative biomass (Wiegand et al., 1974 and Seevers et al., 1973). The correlation coefficients reported ranged from 0.30 for CH7 with crop cover to 0.88 for CH6 with leaf area index. Similar correlations were reported by Tucker (1979).

Ratios of the MSS DCs have been used to estimate and monitor green biomass, etc. (Rouse et al., 1973; 1974; Carneggie et al., 1974; Johnson, 1976, and Maxwell, 1976). The coefficients of determinations were slightly higher than those for the corresponding channel differences. The 12 pairwise ratios (six of which are inverses of the other six) will be denoted by R45 = CH4/CH5, R46 = CH4/CH6, etc.

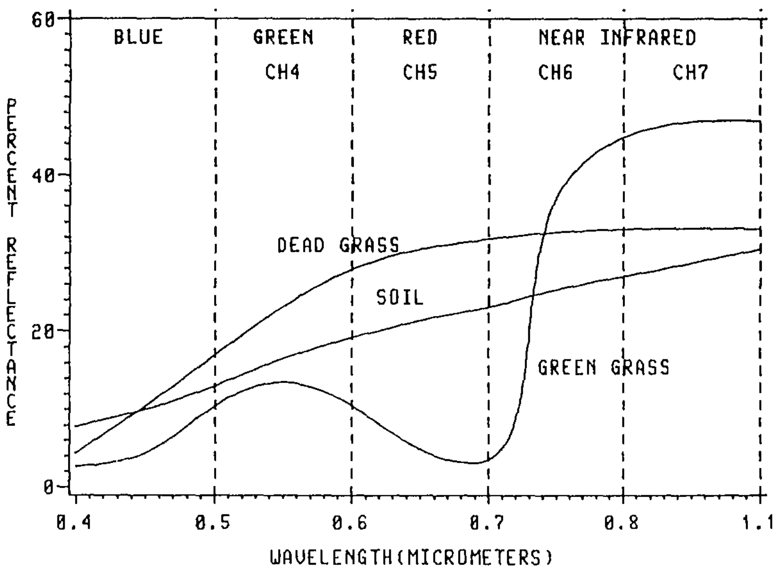


FIGURE 1. Idealized reflectance patterns of herbaceous vegetation and soil from 0.4 to 1.1. mm (Deering et al., 1975).

Rouse et al. (1973) proposed using the normalized difference of DCs from CH7 and CH5 for monitoring vegetation, which will be referred to as ND7. Deering et al. (1975) added 0.5 to ND7 to avoid negative values and took the square root of the result to stabilize the variance. This index is referred to as the transformed vegetation index and will be denoted by TVI7. Similar formulae using CH6 and CH5 were proposed:

$$\mathrm{ND6} = (\mathrm{CH6} - \mathrm{CH5})/(\mathrm{CH6} + \mathrm{CH5}),$$
$$\mathrm{ND7} = (\mathrm{CH7} - \mathrm{CH5})/(\mathrm{CH7} + \mathrm{CH5}),$$
$$\mathrm{TVI6} = (\mathrm{ND6} + 0.5)^{1/2},$$
$$\mathrm{TVI7} = (\mathrm{ND7} + 0.5)^{1/2}.$$

Our experience has been that the addition of 0.5 does not eliminate all negative values. We suggest the following computationally correct formulae:

$$\mathrm{TVI6} = (\mathrm{ND6} + 0.5)/\mathrm{ABS}(\mathrm{ND6} + 0.5) \times [\mathrm{ABS}(\mathrm{ND6} + 0.5)]^{1/2},$$
$$\mathrm{TVI7} = (\mathrm{ND7} + 0.5)/\mathrm{ABS}(\mathrm{ND7} + 0.5) \times [\mathrm{ABS}(\mathrm{ND7} + 0.5)]^{1/2},$$

where ABS denotes absolute value and 0/0 is set equal 1. In Example 1, it is shown that these formulae are equivalent for decision making to the basic ratios R65 and R75. Therefore, their use can only be justified if either they improve the regression fit or they normalize the regression errors (Draper and Smith, 1966).

Kauth and Thomas (1976) used the technique of sequential orthogonalization underlying the Gram–Schmidt process to produce an orthogonal transformation of the original Landsat data space to a new four-dimensional space. They called it the "Tasseled Cap" transformation and named the four new axes brightness (soil brightness index, SBI), greenness (green vegetative Index, GVI), yellow stuff (YVI), and nonsuch (NSI). The names attached to the new axes indicate the characteristics the indices were intended to measure. The coefficients in the following formulae are taken from Kauth et al (1978):

$$\mathrm{SBI} = 0.332\,\mathrm{CH4} + 0.603\,\mathrm{CH5} + 0.675\,\mathrm{CH6} + 0.262\,\mathrm{CH7},$$
$$\mathrm{GVI} = -0.283\,\mathrm{CH4} - 0.660\,\mathrm{CH5} + 0.577\,\mathrm{CH6} + 0.388\,\mathrm{CH7},$$
$$\mathrm{YVI} = -0.899\,\mathrm{CH4} + 0.428\,\mathrm{CH5} + 0.076\,\mathrm{CH6} - 0.041\,\mathrm{CH7},$$
$$\mathrm{NSI} = -0.016\,\mathrm{CH4} + 0.131\,\mathrm{CH5} - 0.452\,\mathrm{CH6} + 0.882\,\mathrm{CH7}.$$

Wheeler et al. (1976) and Misra et al. (1977) applied principal component analysis to MSS DC data. The structure of the resulting transformation and the interpretation of the principal components are similar to those for the Kauth–Thomas transformation:

$$\mathrm{MSBI} = 0.406\,\mathrm{CH4} + 0.600\,\mathrm{CH5} + 0.645\,\mathrm{CH6} + 0.243\,\mathrm{CH7},$$
$$\mathrm{MGVI} = -0.386\,\mathrm{CH4} - 0.530\,\mathrm{CH5} + 0.535\,\mathrm{CH6} + 0.532\,\mathrm{CH7},$$
$$\mathrm{MYVI} = 0.723\,\mathrm{CH4} - 0.597\,\mathrm{CH5} + 0.206\,\mathrm{CH6} - 0.278\,\mathrm{CH7},$$
$$\mathrm{MNSI} = 0.404\,\mathrm{CH4} - 0.039\,\mathrm{CH5} - 0.505\,\mathrm{CH6} + 0.762\,\mathrm{CH7}.$$

The similarity of the Kauth–Thomas and Wheeler–Misra results is remarkable in light of the fact that the ideas and techniques underlying the two processes are quite different. With principal component analysis the experimenter imposes no prior order or physical interpretation on the principal directions. Principal component analysis is in effect a successive factorization of the total variation in the data into mutually orthogonal components, the order being established by the successive directions of maximum variation. Gram–Schmidt orthogonalization, however, gives the experimenter the freedom to indirectly establish a physical interpretation by choosing the order in which the calculations are performed.

Misra et al. (1977) proposed another linear transform, based on the idea of spectral brightness and contrast. Generalizations of spectral brightness and contrast were defined in spectral density space, and then transformed back to count space. The first two components of the resulting transformation are similar to the first two components of the two preceding transformations:

$$\begin{aligned}
\mathrm{SSBI} &= 0.437\,\mathrm{CH4} + 0.564\,\mathrm{CH5} \\
&\quad + 0.661\,\mathrm{CH6} + 0.233\,\mathrm{CH7}, \\
\mathrm{SGVI} &= -0.437\,\mathrm{CH4} - 0.564\,\mathrm{CH5} \\
&\quad + 0.661\,\mathrm{CH6} + 0.233\,\mathrm{CH7}, \\
\mathrm{SYVI} &= -0.437\,\mathrm{CH4} + 0.564\,\mathrm{CH5} \\
&\quad - 0.661\,\mathrm{CH6} + 0.233\,\mathrm{CH7}, \\
\mathrm{SNSI} &= -0.437\,\mathrm{CH4} + 0.564\,\mathrm{CH5} \\
&\quad + 0.661\,\mathrm{CH6} - 0.233\,\mathrm{CH7}.
\end{aligned}$$

Richardson and Wiegand (1977) used the perpendicular distance to the "soil line" as an indicator of plant development. The "soil line," a two-dimensional analogue of the Kauth–Thomas SBI, was estimated by linear regression. Two perpendicular vegetation indices were proposed:

$$\begin{aligned}
\mathrm{PVI7} &= \left[(0.355\,\mathrm{CH7} - 0.149\,\mathrm{CH5})^2 \right. \\
&\quad \left. + (0.355\,\mathrm{CH5} - 0.852\,\mathrm{CH7})^2\right]^{1/2}, \\
\mathrm{PVI6} &= \left[(-0.498 - 0.457\,\mathrm{CH5} \right. \\
&\quad + 0.498\,\mathrm{CH6})^2 \\
&\quad + (2.734 + 0.498\,\mathrm{CH5} \\
&\quad \left. - 0.543\,\mathrm{CH6})^2\right]^{1/2}.
\end{aligned}$$

Evidently a minor error was made in the derivation of PVI6. The formula for PVI6 should be:

$$\begin{aligned}
\mathrm{PVI6} &= \left[(-2.507 - 0.457\,\mathrm{CH5} \right. \\
&\quad + 0.498\,\mathrm{CH6})^2 \\
&\quad + (2.734 + 0.498\,\mathrm{CH5} \\
&\quad \left. - 0.543\,\mathrm{CH6})^2\right]^{1/2}.
\end{aligned}$$

These formulae are computationally inefficient and do not distinguish right from left of the soil line (water from green stuff). The standard formula from analytic geometry for the perpendicular distance from a point to a line solves this difficulty (Salas and Hille, (1978):

$$\begin{aligned}
\mathrm{PVI6} &= (1.091\,\mathrm{CH6} - \mathrm{CH5} - 5.49) \\
&\quad /(1.091^2 + 1^2)^{1/2}, \\
\mathrm{PVI7} &= (2.4\,\mathrm{CH7} - \mathrm{CH5} - 0.01) \\
&\quad /(2.4^2 + 1^2)^{1/2}.
\end{aligned}$$

The difference vegetation index (DVI), suggested by Richardson and Wiegand (1977) as computationally easier than PVI7, is essentially a rescaling of PVI7:

$$\mathrm{DVI} = 2.4\,\mathrm{CH7} - \mathrm{CH5}.$$

The Ashburn vegetation index (Ashburn, 1978) was suggested as a measure of green growing vegetation. The doubling of CH7 is to make the scale compatible: CH7 is 6-bit data and has one-half the range of the other three bands, which are 8-bit data:

$$\mathrm{AVI} = 2.0\,\mathrm{CH7} - \mathrm{CH5}.$$

Hay et al. (1979) proposed a vegetation indicator called greenness above bare soil (GRABS). This was an attempt to develop an indicator for which a threshold value could be specified for detecting green vegetation. The calculations were made using the Kauth–Thomas tassel cap transformation applied to sun-angle and haze-corrected data. The resulting index is quite similar to the GVI, since the contribution of SBI is less than 10% of GVI:

$$\mathrm{GRABS} = \mathrm{GVI} - 0.09178\,\mathrm{SBI} + 5.58959.$$

Kanemasu et al. (1977) regressed winter wheat leaf area measurements of MSS band ratios and produced the following regression equation:

$$\begin{aligned}\mathrm{ELAI} = {} & 2.68 - 3.69\,\mathrm{R45} - 2.31\,\mathrm{R46}\\ & + 2.88\,\mathrm{R47}\\ & + 0.43\,\mathrm{R56} - 1.35\,\mathrm{R57}\\ & + 3.07[\mathrm{R45} - (0.5\,\mathrm{R47})(\mathrm{R45})].\end{aligned}$$

Pollack and Kanemasu (1979) later used a larger data set plus stepwise regression and obtained another regression equation.

$$\begin{aligned}\mathrm{CLAI} = {} & 0.366 - 2.265\,\mathrm{R46}\\ & - 0.431(\mathrm{R45} - \mathrm{R47})(\mathrm{R45})\\ & + 1.745\,\mathrm{R45} + 0.57\,\mathrm{PVI7}.\end{aligned}$$

Separate regression equations were also obtained for CLAI values above and below 0.5:

$$\begin{aligned}\mathrm{LAI} = {} & 1.903 - 1.138\,\mathrm{R56}\\ & - 0.071(\mathrm{R45} - \mathrm{R47})\mathrm{R45}\\ & - 0.016\,\mathrm{PVI6},\end{aligned}$$

if CLAI is less than 0.5,

$$\mathrm{LAI} = -5.33 + 0.036\,\mathrm{PVI7} + 6.54\,\mathrm{TVI6},$$

if CLAI is greater than 0.5.

Thompson and Wehmanen (1979) proposed a technique utilizing transformed DC data for detection of agricultural vegetation undergoing moisture stress. The MSS data are rotated into the Kauth–Thomas vectors (GVI, SBI, YSI, NSI) to screen out clouds, water, bare soil, etc. Each vector is evaluated and any vector having values considered unreasonable for agricultural data is discarded. The remaining pixels are considered the good pixels. 1% of the pixels with the lowest GVI values are then discarded. The lowest GVI value remaining becomes the soil line. A green number is then computed for each pixel by subtracting the soil line from GVI. The green index number (GIN) is then an estimate of the percentage of pixels in the scene with a green number greater than or equal to 15:

$$\mathrm{GIN} = \frac{\text{number of pixels with a green number of } 15+}{\text{number of good pixels}} \times 100.$$

Empirical Relationships among Vegetative Indices

Richardson and Wiegand (1977) correlated eight VIs (GVI, DVI, SBI, PVI6, PVI7, TVI6, TVI7, and R57) with four plant component variables (crop cover, shadow cover, plant height, and leaf area index). The correlation coefficients obtained by plant component with the VIs (excluding SBI) were very similar. Later, Wiegand et al. (1979) correlated leaf area indices for winter wheat fields to five VIs (TVI7, TVI6, PVI7, PVI6, and GVI). The correlation coefficients within and among fields were similar.

Aaronson et al. (1979) studied the similarities and differences among several VIs (AVI, DVI, GVI, PVI7, TVI7, and KVI). The obtained correlation coefficients ranged from 0.8 to 1.0 and were stable from spring greenup to harvest. Aaronson and Davis (1979) later used a large data set, which included vegetation measurements and several VIs, to study interrelationships. The VIs (AVI, DVI, GVI, KVI, PVI6, PVI7, TVI6, and TVI7) were correlated against each other and against vegetation measures such as plant height from tillering through harvest. The correlation coefficients between the VIs ranged from 0.81 to 1.00, and those between VIs and vegetation measures mostly cluster around 0.7.

Lautenschlager and Perry (1981) studied the empirical relationships among the VIs listed in the above section using cluster analysis. The absolute value of the bivariate correlations was used as the measure of distance between VIs, and the average distance between elements was used as the between-cluster distance. This procedure separated the VIs into two large groups plus a number of small groups. One large group contained VIs based on CH5 and CH7, which included AVI, PVI7, R75, TVI7, and ND7. The other large group contained VIs, based on CH5 and CH6, and a few VIs involving three or all four channels, which included GRABS, CLAI, R65, TVI6, ND6, GVI, MGVI, PVI6, and SGVI. The VIs within these two groups had absolute simple correlations greater than 0.90, with most greater than 0.95. The elements of these two large groups were correlated at 0.8 or higher. Three smaller groups readily apparent were: (NSI, R76), (R64, R74), and (SBI, MSBI, SSBI, SNSI).

An Equivalence Relation for Spectral Vegetation Indices

In this section, a definition of VI equivalence is developed. The utility of this definition is demonstrated by examples in the context of alarm models and graphical display. Vegetation indices are functions which associate a real number to each four-dimensional MSS DC vector, (CH4, CH5, CH6, CH7). To give a precise statement of vegetation index equivalence it is convenient to employ standard function notation: f: $S_1 \rightarrow S_2$ denotes a function from the set S_1 into the set S_2; $f(X)$, the value of f at the point (X) of S_1; Dom(f), the domain of f; Ran(f), the range of f; and f^{-1}: $S_2 \rightarrow S_1$, the inverse of f when it exists. The inverse exists if, and only if, f is one-to-one and onto. The composition of two functions has an inverse if, and only if, both functions have inverses, in which case $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$. The reader unfamiliar with this notation may wish to study Figs. 2–5 before proceeding to the formal presentation that follows. A short explanation of the function notation is given in the Appendix.

It might seem that VI equivalence should correspond to function equality; i.e., $V_1 = V_2$ if, and only if, $V_1(X) = V_2(X)$ for each MSS DC vector X. However, this requirement is too restrictive because it requires that both VIs have the same graph and ignores the decisions made on the basis of the VI values. Since vegetation indices are formulae used in making decisions about crop characteristics and conditions, it is appropriate to say that two VIs are equivalent if, and only if, the same decision results regardless of the VI employed. This means that two VIs, V_1 and V_2, are equivalent for making the set of decisions D if, and only if, for every decision rule d_1: $\mathrm{Ran}(V_1) \to D$, there corresponds a decision rule d_2: $\mathrm{Ran}(V_2) \to D$ such that the decision, based on d_2 and V_2, is the same as the decision based on d_1 and V_1 for all MSS DC vectors X; that is, $d_1(V_1(X)) = d_2(V_2(X))$ for each X. It is easy to see that two vegetation indices, V_1 and V_2, are equivalent if, and only if, there exists a one-to-one onto function T: $\mathrm{Ran}(V_1) \to \mathrm{Ran}(V_2)$ such that $T \circ V_1 = V_2$. Thus the same decision results regardless of the VI used; that is,

$$V_1^{-1}[T^{-1}(d)] = (T \circ V_1)^{-1}(d) = V_2^{-1}(d) \tag{1}$$

for each decision d in D, where the superscript -1 indicates the inverse image of d under the given function. The relationship defined is an equivalence relation on the set of vegetation indices; that is,

(i) Each VI is equivalent to itself: reflexive property.
(ii) If V_1 is equivalent to V_2, then V_2 is equivalent to V_1: symmetric property.
(iii) If V_1 is equivalent to V_2 and V_2 is equivalent to V_3, then V_1 is equivalent to V_3: transitive property.

Many tedious computations are avoided by using these properties.

A number of studies have investigated the transformed vegetation indices TVI6 and TVI7 and the corresponding ratios R65 and R75 as predictors of biomass, leaf area, plant height, and percent cover. The predictive ability of TVI6 and R65 or TVI7 and R75 are similar as evidence by the estimated correlation coefficient. We now show that the transformed vegetation index and its generalizations are equivalent to the corresponding ratios. The sequence of examples that follows will make clear not only the algebraic and geometric meanings of VI equivalence but also demonstrate the utility and appropriateness of this definition.

Example 1

Let a and b be positive constants, and define the functions f, g, and T by

$$f(X_5, X_7) = (aX_7 - bX_5)/(aX_7 + bX_5),$$
$$g(X_5, X_7) = X_7/X_5,$$
$$T(y) = (b/a)[(1+y)/(1-y)]$$

for X_5 and X_7 positive and $\mathrm{ABS}(y)$ less than one. Observe that T is invertible; in fact,

$$T^{-1}(z) = (az - b)/(az + b)$$

for z positive.

Thus, f and g are equivalent and the values of f can be computed from the values of g and vice versa:

$$(T \circ f)(X_5, X_7) = g(X_5, X_7),$$
$$(T^{-1} \circ g)(X_5, X_7) = f(X_5, X_7).$$

The relationship between ND7 and R75 is illustrated in Fig. 2. The important

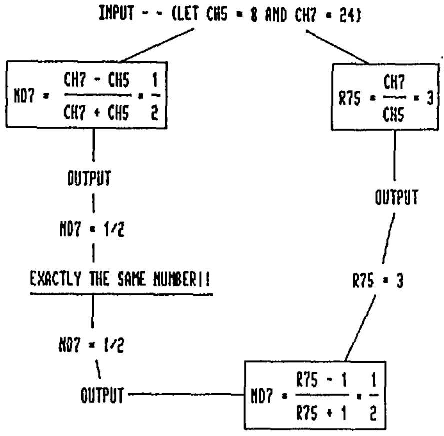


FIGURE 2. The equivalence of vegetation indices means the value of one index can be computed from the value of the other index. The flow chart outlines the computation of ND7 from R75.

point is: Knowing the value of one index is equivalent to knowing the value of the other index—the indices are therefore equivalent for decision making.

The equivalence of TVI7 and R75 is shown as follows: Let k and p be real, and define the functions G: $(-1,1) \to (k-1, k+1)$ and H: $(k-1, k+1) \to (L, U)$ by

$$G(v) = v + k,$$

$$H(w) = w[\mathrm{ABS}(w)]^{p-1}$$

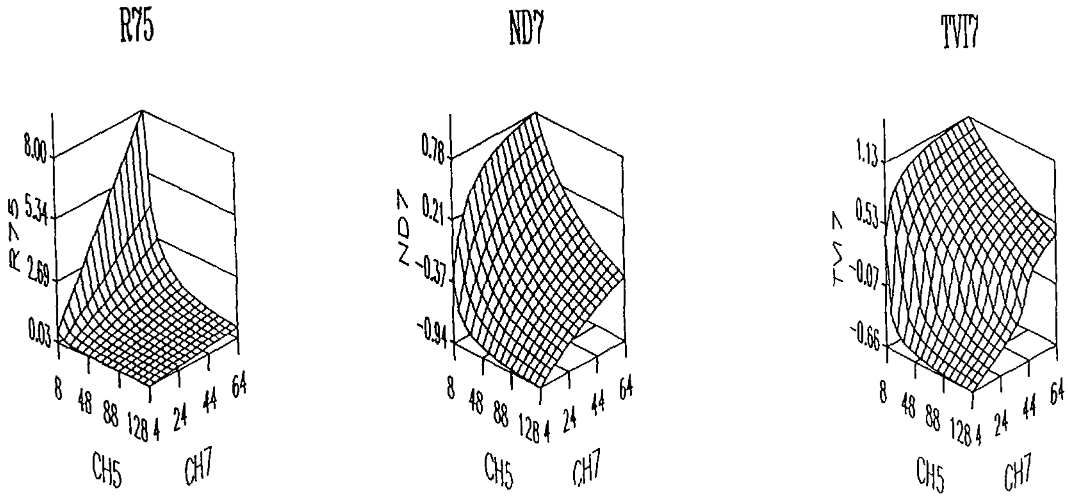
for w between $k-1$ and $k+1$, $L = (k-1)[\mathrm{ABS}(k-1)]^{p-1}$, $U = (k+1)[\mathrm{ABS}(k+1)]^{p-1}$, for $\mathrm{ABS}(v) < 1$ and $0/0$ defined as 1. It is easy to verify that G and H are one-to-one and onto and that

$$(H \circ G \circ T^{-1} \circ g)(X_5, X_7) = (f(X_5, X_7) + k) \times [ABS(f(X_5, X_7) + k)]^{p-1}.$$

Taking $k = p = 1/2$ and $a = b = 1$ yields a one-to-one function between TVI7 and R75:

$$(H \circ G \circ T^{-1})R75 = TVI7.$$

Another way to view VI equivalence is that equivalent VIs divide the DC space into the same set of equivalence classes. This interpretation is illustrated graphically in Fig. 3.



FIGURES 3(a), 3(b), and 3(c). The response surfaces of R75, ND7, and TVI7 determine the same equivalence classes—the set of lines emanating from the origin in the two-dimensional DC space.

Example 2

This example illustrates the utility of VI equivalence in the context of alarm models. Suppose we take as our decision rule:

• Sound a "warning bell" if ND7 is above

$$B = 0.5.$$

Using the relationships developed in Example 1, it is easy to see that equivalent decision rules based on R75 and TVI7 are:

• Sound a "warning bell" if R75 is above

$$A = (1 + B)/(1 - B) = 3.0.$$

• Sound a "warning bell" if TVI7 is above

$$C = \sqrt{\mathrm{ABS}(B) + 0.5} = 1.$$

Applying the hypothetical alarm model to spectral data taken in 1980–81 over a winter wheat field in Wilbarger County, Texas, one sees that precisely the same action is taken regardless of the decision rule used (Fig. 4). The bell rang from 17 November through 10 January.

Example 3

As a last illustration, VI equivalence is examined in the context of false color display of digital spectral data. Figures 5(a) and 5(b) show two false color images produced from two channels of multispectral data. The color assignments in Fig. 5(a) are made using a ratio of the two channels; the intervals associated with the different colors are indicated in the attached scale. The color assignments in Fig. 5(b) are made using the normalized difference of the same two channels; as before, the intervals associated with the different colors are indicated in the attached scale.

Clearly, the same information is not displayed in both images. One might ask the question: Which index is superior for displaying this type of data? Neither! This can be reasoned as follows. Using the relationships developed in Example 1, the same color assignments can be made in Fig. 5(a) using the normalized difference. The interval divisions associated with the

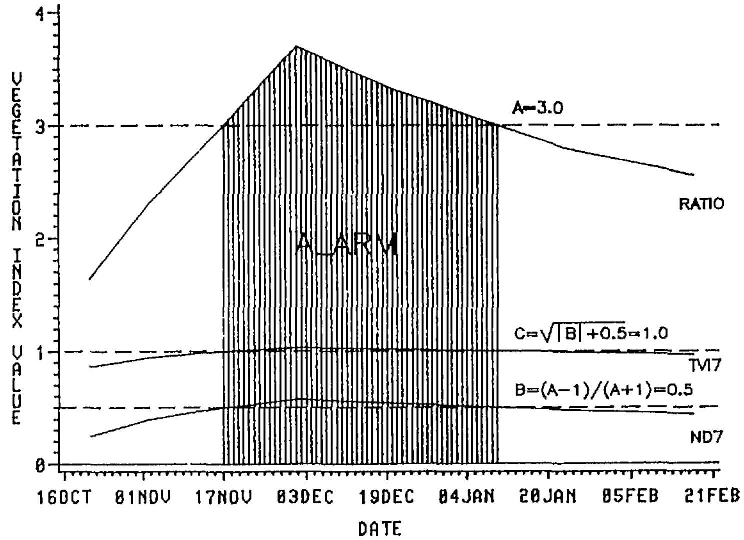


FIGURE 4. The hypothetical alarm model shows that no matter which VI is used exactly the same decision will be made. This illustrates that VI's can be equivalent for decision making and not have the same graph.

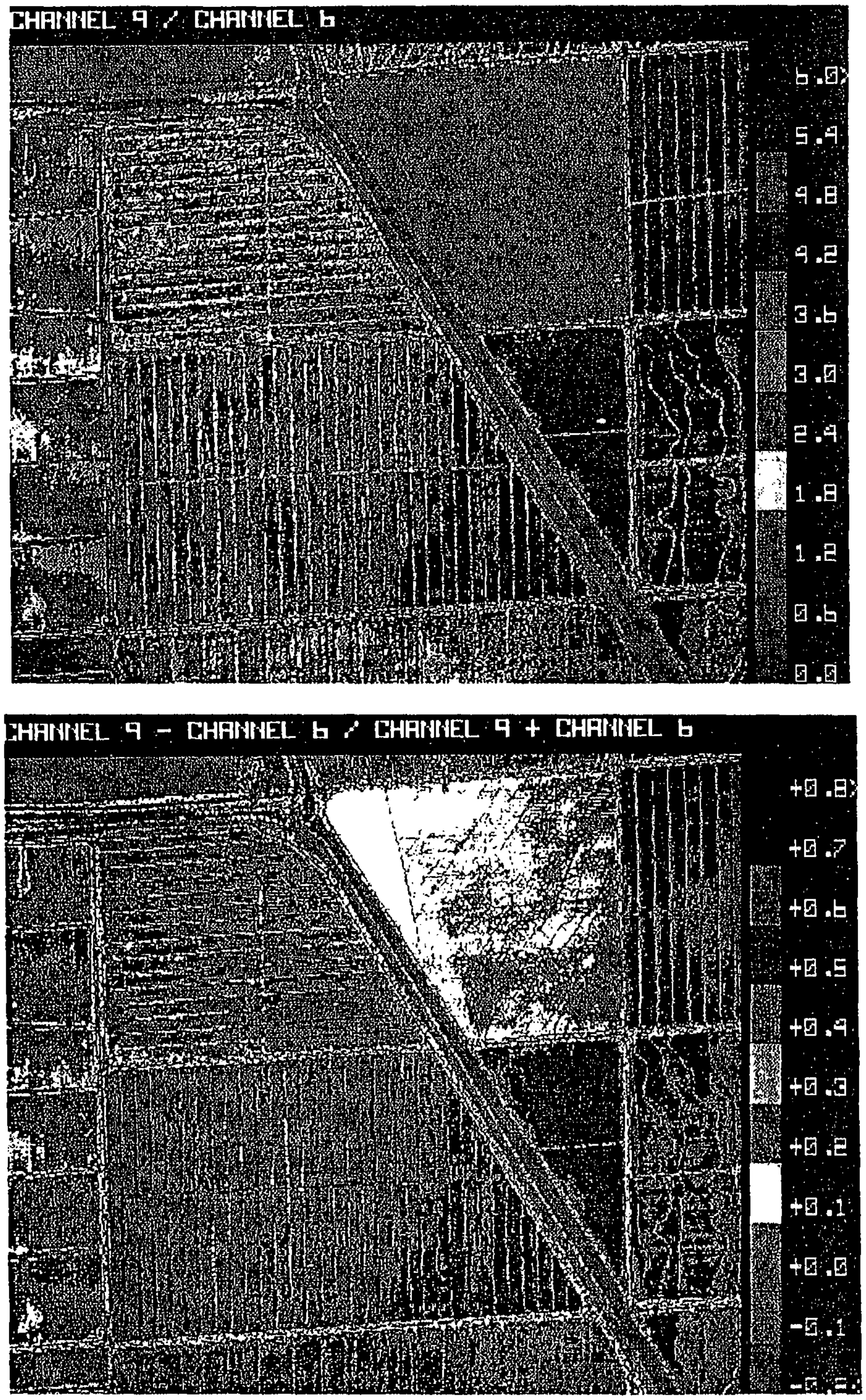


FIGURE 5. The two images look very different. This difference results from the color assignment intervals rather than from the VI used. The top image can be produced from the normalized difference, and the bottom image can be produced from the ratio by using the color assignment intervals specified in Table 1.

TABLE 1 Interval Division for Producing the False Color Image.

Interval Divisions			
Fig. 5(A)		Fig. 5(B)	
Using Ratio	Using Normalized Difference	Using Normalized Difference	Using Ratio
6.0	0.86	0.8	9.00
5.4	0.69	0.7	5.67
4.8	0.66	0.6	4.00
4.2	0.62	0.5	3.00
3.6	0.57	0.4	2.33
3.0	0.56	0.3	1.86
2.4	0.41	0.2	1.50
1.8	0.29	0.1	1.22
1.2	0.09	0.0	1.00
0.6	−0.25	−0.1	0.82
0.0	−1.00	−0.2	0.67
Assuming the ratio, the corresponding normalized difference is $\mathrm{ND} = \frac{\text{ratio} - 1}{\text{ratio} + 1}$		Assuming the normalized difference, the corresponding ratio is $\text{ratio} = \frac{1 + \mathrm{ND}}{1 - \mathrm{ND}}$	

normalized difference are given in column 2 of Table 1. Similarly the color assignments in Fig. 5(b) can be made using the ratio and the interval division given in column 4 of Table 1. Thus, it is not a question of which index is superior or more sensitive. It is how the interval divisions are chosen. The authors are indebted to Dr. Ray Jackson (USDA/ARS, Phoenix, Arizona) and Mr. John Millard (Ames Research Center, Moffett Field, California) for the color imagery used in this example.

Summary and Conclusions

Since the launch of Landsat 1 in 1972, investigators have derived numerous formulae for the reduction of multispectral scanner measurements to a single value for predicting and assessing vegetation characteristics such as species, leaf area, stress, and biomass. Part I of this paper summarized many of these formulae and the empirical relationships among them. Most formulae fall into one of two basic categories: those that use ratios or those that use differences to exploit the spectral characteristics of soil and vegetation. Part II of this paper developed the idea of two vegetation indices being equivalent: two indices were taken to be equivalent, if the decision made on the basis of one index could have equally well been made on the basis of the other index. The significance of this idea was studied by example in several contexts, and it was shown that for all practical purposes several widely used indices are equivalent.

Appendix. Modern Function Notation

Almost as basic to modern mathematics as the concept of a set is the concept of a function. If A and B are sets, a function f from A into B is a rule which associates with each element x of A an element $f(x)$ of B. If f is a function from A to B, one writes f: $A \to B$. If $x \in A$, then $f(x)$ is called the value of f at x. Often functions are thought of as transforms or maps from one set into another. In today's terminology the terms "transformation," "mapping," and "operator" are synonomous with function. If f: $A \to B$ and $x \in A$, then one says that f maps x to $f(x)$. An example illustrate these concepts. Let A be any nonempty set. The function f: $A \to A$ defined by $f(a) = a$ for all $a \in A$ is called the identity function and is denoted i_A.

If f: $A \to B$ is a function, the set A is called the domain of f and the set B is called the range of f. A function is defined by specifying its value for each element belonging to its domain. Two functions f and g from A to B are said to be equal if $f(x) = g(x)$ for all $x \in A$.

Suppose that g: $A \to B$ and f: $B \to C$ are functions. If $x \in A$, then we may define a function from A into C by first mapping x to $f(x)$ and then mapping $f(x)$ to $f(g(x))$. This function is called the composite of f and g and is denoted by $f \circ g$. According to the above definitions

$$(f \circ g)x = f(g(x)).$$

Let f: $A \to B$ be a function. Then f is said to be one-to-one if, $f(x) = f(y)$ implies $x = y$. For example, if f: $R \to R$ is the function $f(x) = 5x + 3$, then f is one-to-one, because $5x + 3 = 5y + 3$ implies that $x = y$. The function $g(x) = x^2$ is not one-to-one, since $g(3) = g(-3)$.

A function f: $A \to B$ is said to be onto if, for every $y \in B$, there exists an $x \in A$ such that $f(x) = y$. For example, $f(x) = 5x + 3$ is onto, since for every $y \in R$ one has $f((y-3)/5) = y$. However, $g(x) = x^2$ is not onto, since there does not exist a real number x such that $f(x) = -1$.

If f: $A \to B$ is one-to-one and onto, then for every $b \in B$, there is exactly one $a \in A$ such that $f(a) = b$. Therefore, one may define the function f^{-1}: $B \to A$ by $f^{-1}(b) = a$. The function f^{-1} is called the inverse of f. Clearly, one has

$$f \circ f^{-1} = i_B$$

and

$$f^{-1} \circ f = i_A.$$

If f: $A \to B$ and C is a subset of B, the set $[a \in A | f(a) \in C]$ is called the inverse image of C, and is denoted by $f^{-1}[C]$. The collection of all inverse images of the singleton set $[b]$, as b ranges over B, partitions the set A into mutually disjoint sets. The individual members of this collection are called equivalence classes. For example, if r: $R^+ \times R^+ \to R$ is the function $r(x, y) = x/y$, the equivalence class $f^{-1}[b] = [(x, y) \quad x/y = b]$ is the line emanating from the origin and having slope b^{-1}.

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